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High-Frequency Sum Rules for Classical One-Component Plasma in a Magnetic Field

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A high-frequency sum-rule expansion is derived for all elements of a classical plasma dielectric tensor in the presence of an external magnetic field. Ω_4^{13} is found to be the only coefficient of ω^{-4} that has no correlational and finite-radiation-temperature contributions. The finite-radiation-temperature effect results in an upward renormalization of the frequencies of the modes; it also leads to either reduction of the negative correlational effect on the positive thermal dispersion or, together with correlation, enhancement of the positive thermal dispersion for finite k, depending on the direction of propagation. Further, for the extraordinary mode, the finite-radiation-temperature effect increases the positive refractive dispersion for finite k.

1. INTRODUCTION

The high-frequency sum rule provide the coefficient of the inverse powers of ω in the high-frequency asymptotic expansions of $\boldsymbol{\alpha}(\mathbf{k})$. Sum rules follow from the equations of motion (i.e., conservation laws) via the fluctuation-dissipation theorems (FDT) and the Kramers–Kronig (KK) relations for the response functions (Kalman, 1978).

In this work we consider an anisotropic system in the presence of an external magnetic field, since the result for an isotropic system is already known (Kalman and Genga, 1986). In this system the dielectric tensor has six independent elements. Also, the relationship between the elements of the external and current–current response function and the elements of the dielectric tensor become quite involved.

The high-frequency expansion is carried out to order ω^{-5}. The method of derivation is similar to the standard approach (de Gennes, 1959) and relies heavily on the Hamiltonian formalism. It is known (Kalman and

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Genga, 1986) that in order to describe the transverse interaction, the particle Hamiltonian has to be enlarged to include the photon degrees of freedom. In so doing we encounter, in addition to the particle contribution to the sum-rule coefficients, the photon gas coexistent with the high-temperature plasma, which generates its own contribution. As in the magnetic field-free case (Kalman and Genga, 1986), the evaluation of the contribution is hampered by two circumstances; the first is the well-known classical ultraviolet divergence, which requires that even within the framework of a classical theory one describe the photons via the quantum Bose–Einstein distribution, while the second difficulty arises from the fact that the equilibrium description implies the existence of one single temperature for the combined particle–photon system. Such an equilibrium, however, seldom prevails in any but astrophysical situations. Thus, a reasonable *ad hoc* approximation, described in Section 2, is used to decouple the photons from the particle system.

In Section 2 we derive the general relationships between the current-current response function sum-rule coefficients and those of the dielectric tensor. Then we calculate the exact ω^{-2}, ω^{-3}, ω^{-4}, and ω^{-5} sum-rule coefficients for all elements of the dielectric tensor, respectively. In Section 3 we calculate the long-wavelength limit of the results. We use the same procedure as that given in the Appendix of Kalman and Genga (1986) for the magnetic field-free case in solving the lengthy algebra leading to the results of Section 2. In Section 4 we determine strong coupling and finite-radiation-temperature effects on the high-frequency modes, i.e., the plasma mode and the high-frequency extraordinary mode for propagation parallel and perpendicular to the magnetic field, respectively.

2. SUM RULES FOR FULL DIELECTRIC TENSOR

In this case a quantity of central importance is $Q^{\mu\nu}(\mathbf{k}\omega)$, the Fourier transform of the two-point current density correlation function defined by

$$Q^{\mu\nu}(\mathbf{k}\omega)=\frac{1}{2\pi}\int d\tau\, e^{i\omega\tau}Q^{\mu\nu}(\mathbf{k}\tau) \tag{1}$$

where

$$Q^{\mu\nu}(\mathbf{k}\tau)=\frac{e^2}{v}\langle j^{\mu}_{\mathbf{k}}(\tau)j^{\nu}_{-\mathbf{k}}(0)\rangle \tag{2}$$

When we apply the linear fluctuation-dissipation theorem to the current density $j_{\mathbf{k}}$, we find that

$$\hat{\sigma}^{A}_{\mu\nu}(\mathbf{k}\omega)=\frac{4\pi e^2}{\omega}\,\beta_p Q^{\mu\nu}(\mathbf{k}\omega) \tag{3}$$

where the superscript A stands for "anti-Hermitian part of." The frequency moments of $\hat{\alpha}^H_{\mu\nu}(\mathbf{k}\omega)$, $\Omega^{\mu\nu}_{\tau+1}(\mathbf{k})$, are then given by

$$\hat{\Omega}^{\mu\nu}_{l+1}(k) = \frac{4\pi e^2}{V}\beta_p\left(i\frac{d}{d\tau}\right)^{\iota-1}\langle j^{\mu}_{(\mathbf{k})}(\tau)j^{\nu}_{-\mathbf{k}}(0)\rangle|_{\tau=0} \tag{4}$$

where

$$j^{\mu}_{\mathbf{k}} = \sum_i V^{\mu}_i \exp(-i\mathbf{k}\cdot\mathbf{x}_i) \tag{5}$$

with $\mathbf{V}_i$ the velocity of the ith particle. Both odd and even moments exist in equation (4). Odd moments originate from the imaginary part of $\hat{\alpha}^H_{\mu\nu}(\mathbf{k}\omega)$; the even ones originate from the real part of $\hat{\alpha}^H_{\mu\nu}$; $\hat{\alpha}^H$ and $\hat{\Omega}$ are interrelated, i.e.,

$$\hat{\alpha}^H_{\mu\nu}(\mathbf{k}\omega) = -\sum_{\substack{\iota=1\\ \iota\ \text{odd}}}\frac{\hat{\Omega}^{\mu\nu}_{\iota+1}(\mathbf{k})}{\omega^{\iota+1}} \tag{6}$$

$$\hat{\alpha}^H_{\mu\nu}(\mathbf{k}\omega) = -\sum_{\substack{\iota=2\\ \iota\ \text{even}}}\frac{\hat{\Omega}^{\mu\nu}_{\iota+1}(k)}{\omega^{\iota+1}} \tag{7}$$

where the superscript H stands for "Hermitian part of" and prime and double prime denote "real part of" and "imaginary part of," respectively. As a result of equations (5) and (6), the complete high-frequency expansion of the dielectric tensor $\varepsilon_{\mu\nu}(\mathbf{k}\omega)$ and the complete high-frequency expansion of the polarization tensor $\alpha_{\mu\nu}(\mathbf{k}\omega)$ are expressed in a way similar to that of the corresponding "external" $\hat{\alpha}_{\mu\nu}(\mathbf{k}\omega)$, since it is known (Kalman, 1978; Kalman and Genga, 1986) that

$$\begin{aligned} \boldsymbol{\alpha} &= \hat{\boldsymbol{\alpha}}(\boldsymbol{\Delta}-\hat{\boldsymbol{\alpha}})^{-1}\boldsymbol{\Delta}, \qquad \boldsymbol{\Delta} = 11 - n^2\mathbf{T} \\ n &= kc/\omega, \qquad \mathbf{T} = \mathbf{k}\cdot\mathbf{k}/k^2 \end{aligned} \tag{8}$$

with $\Omega^{\mu\nu}_{\iota+1}(\mathbf{k})$ replacing $\hat{\Omega}^{\mu\nu}_{\iota+1}(\mathbf{k})-s$. The relationships between the two sets of coefficients up to $\iota=4$ are (Kalman and Genga, 1986)

$$\begin{aligned} \Omega^{\mu\nu}_2(\mathbf{k}) &= \hat{\Omega}^{\mu\nu}_2(\mathbf{k}) \\ \Omega^{\mu\nu}_3(\mathbf{k}) &= \hat{\Omega}^{\mu\nu}_3(\mathbf{k}) \\ \Omega^{\mu\nu}_4(\mathbf{k}) &= \hat{\Omega}^{\mu\nu}_4(\mathbf{k}) - \hat{\Omega}^{\mu\alpha}_2(\mathbf{k})\hat{\Omega}^{\alpha\nu}_2(\mathbf{k}) \\ \Omega^{\mu\nu}_5(\mathbf{k}) &= \hat{\Omega}^{\mu\nu}_5(\mathbf{k}) - \hat{\Omega}^{\mu\alpha}_2(\mathbf{k})\hat{\Omega}^{\alpha\nu}_3(\mathbf{k}) - \hat{\Omega}^{\mu\alpha}_3(\mathbf{k})\hat{\Omega}^{\alpha\nu}_2(\mathbf{k}) \end{aligned} \tag{9}$$

As discussed in the introduction, the Hamiltonian appropriate for the description of the interaction of the plasma with the transverse electromagnetic field must include the photon degrees of freedom (Kalman and Genga, 1986). Thus, we have

$$H = \frac{m}{2} \sum_i V_i^2 + \frac{1}{2} \sum_{\substack{ij \\ i \neq j}} V(\mathbf{x}_i - \mathbf{x}_j) + \frac{1}{2} \sum_q (e_\mathbf{q} \cdot \mathbf{e}_\mathbf{q} + q^2 c^2 a_\mathbf{q} \cdot \mathbf{a}_\mathbf{q}) \tag{10}$$

with

$$\mathbf{V}_i = \frac{1}{m} \mathbf{P}_i - \frac{e}{m} \left(\frac{4\pi}{V} \right)^{1/2} \sum_\mathbf{q} a_q \exp(i\mathbf{q} \cdot \mathbf{x}_i) - \frac{e}{mc} \bar{A}^0(\mathbf{x}_i) \tag{11}$$

where $\bar{A}^0(\mathbf{x}_i)$ is the vector potential of the ith particle.

We now turn to the calculation of the frequency moments up to $\iota = 4$. In the presence of an external magnetic field, the system is anisotropic and $\hat{\boldsymbol{\alpha}}$ is nondiagonal. The real diagonal and off-diagonal elements satisfy symmetric condition

$$\Omega_{\iota+1}^{\mu\nu}(\mathbf{k}) = \Omega_{\iota+1}^{\nu\mu}(k) \tag{12}$$

whereas the imaginary off-diagonal elements satisfy the antisymmetric condition

$$\Omega_{\iota+1}^{\mu\nu}(\mathbf{k}) = -\Omega_{\iota+1}^{\nu\mu}(\mathbf{k}) \tag{13}$$

The first moment is trivial,

$$\begin{aligned} \hat{\Omega}_2^{\mu\nu}(\mathbf{k}) &= \frac{4\pi e^2}{V} \beta_p \langle j_\mathbf{k}^\mu(0) \mathbf{j}_\mathbf{k}^\nu(0) \rangle \\ &= \frac{4\pi e^2}{V} \beta_p N \frac{m}{\beta_p} \delta^{\mu\nu} \\ &= \omega_p^2 \delta^{\mu\nu} \end{aligned} \tag{14}$$

The second moment yields

$$\begin{aligned} \hat{\Omega}_3^{\mu\nu}(k) &= i \frac{4\pi e^2}{V} \beta_p \frac{d}{d\tau} \langle j_\mathbf{k}^\mu(\tau) \mathbf{j}_\mathbf{k}^\nu(0) \rangle \big|_{\tau=0} \\ &= i \frac{4\pi e^2}{2V} \beta_p [\langle \dot{j}_\mathbf{k}^\mu(\tau) \mathbf{j}_\mathbf{k}^\nu(0) \rangle - \langle j_\mathbf{k}(0) \dot{\mathbf{j}}_\mathbf{k}^\nu(\tau) \rangle]_{\tau=0} \\ &= i \frac{4\pi e^2}{2C} \beta_p \sum_{ij} \langle (\dot{V}_i^\mu V_j^\nu - i\mathbf{k}^\alpha V_i^\alpha V_i^\mu V_j^\nu \\ &\quad - \dot{V}_j^\nu V_i^\mu - ik^\alpha V_j^\alpha V_j^\nu V_i^\mu) \exp[-i\mathbf{k} \cdot (x_i - \mathbf{x}_j)] \rangle \\ &= i \frac{4\pi e^3}{m^2 c} \frac{N}{V} \varepsilon^{\mu\nu\rho} B^{0\rho} \\ &= i \frac{\omega_p^2 e}{mc} \varepsilon^{\mu\nu\rho} B^{0\rho} \end{aligned} \tag{15}$$

The second step in equation (15) is true because of the time translation invariance of

$$\langle j_{\mathbf{k}}^{\mu}(\tau)\mathbf{j}_{\mathbf{k}}^{\nu}(0)\rangle$$

$\dot{V}_i^{\mu}$ is the acceleration of the ith particle in the μ direction, given by

$$\dot{V}_i^{\mu} = -\frac{1}{m}\frac{\partial\Phi}{x_i^{\mu}} + \frac{e}{m}\left(\frac{4\pi}{V}\right)^{1/2} \sum_{\mathbf{q}} e_{\mathbf{q}}^{\mu} \exp(i\mathbf{q}\cdot\mathbf{x}_i)$$
$$+\frac{ie}{m}\left(\frac{4\pi}{V}\right)^{1/2} \sum_{q} [\bar{V}_i x(q\times a)]^{\mu} \exp(i\mathbf{q}\cdot\mathbf{x}_i) + \frac{e}{mc}(V_i \times B^0)^{\mu} \tag{16a}$$

where

$$\frac{\partial\Phi}{\partial x_i^{\mu}} = \frac{\partial H}{\partial x_i^{\mu}} - \frac{\partial R}{\partial x_i^{\mu}}$$
$$R = \frac{m}{2}\sum_m V_m^2, \qquad \Phi = \tfrac{1}{2}\sum_{\substack{mn \\ m\neq n}}^{m} V(\mathbf{x}_m - \mathbf{x}_n) \tag{16b}$$

H is the total Hamiltonian defined by equation (10). The third moment leads to

$$\Omega_4^{\mu\nu}(\mathbf{k}) = \frac{4\pi e^2}{V}\beta_p\left(i\frac{d}{d\tau}\right)^2 \langle j_{\mathbf{k}}^{\mu}(\tau)\mathbf{j}_{\mathbf{k}}^{\nu}(0)\rangle|_{\tau=0}$$
$$= \frac{4\pi e^2}{V}\beta_p \langle \dot{\mathbf{j}}_{\mathbf{k}}^{\mu}(0)\dot{\mathbf{j}}_{\mathbf{k}}^{\nu}(0)\rangle$$
$$= \frac{4\pi e^2}{V}\beta_p \sum_{ij}\langle(\dot{V}_i^{\mu}\dot{V}_i^{\nu} + k^{\alpha}k^{\delta}V_i^{\alpha}\dot{V}_j^{\nu}V_i^{\mu} - ik^{\alpha}V_i^{\alpha}V_j^{\nu}$$
$$+ ik^{\alpha}V_j^{\delta}V_j^{\nu}\dot{V}_i^{\mu})\exp[-i\mathbf{k}\cdot(\mathbf{x}_i - \mathbf{x}_j)]\rangle \tag{17}$$

The four terms in equation (17) can be evaluated by a series of operations based on the canonical equations. Details of the calculation are similar to the one given in Kalman and Genga (1986). As already known (Kalman and Genga, 1986), the presence of the photon degrees of freedom $a_{\mathbf{q}}^{\mu}$ and $e_{\mathbf{q}}^{\mu}$ lead to the presence of averages of field coordinates of the type $\langle a_{\mathbf{q}}^{\mu}\mathbf{a}_{\mathbf{q}}^{\mu}\rangle$ and $\langle e_{\mathbf{q}}^{\mu}\mathbf{e}_{\mathbf{q}}^{\mu}\rangle$, which are expressible in terms of the inverse temperature β in strict thermal equilibrium. In such a situation we have two distinct, particle and radiation, temperatures, represented by β_p and β_r, respectively, for the system. Further, the evaluation of field coordinate averages quote above classically leads to divergent integrals when summation is over the possible $\mathbf{q}$ modes of the electromagnetic field, which is also in agreement with the classical ultraviolet divergence of the electromagnetic field energy. Therefore, such averages must be evaluated quantum mechanically even in the framework of a classical theory such as the one we are considering. As in the magnetic field-free case (Kalman and Genga, 1986), if we introduce

$$C^i_{\mathbf{q}} = 2^{-1/2}[\omega_{\mathbf{q}}^{1/2} a^{\mu}_{\mathbf{q}} + (i/\omega_q^{1/2}) e^{\mu}_{\mathbf{q}}]\varepsilon^{\mu i} \tag{18}$$

$$\omega_{\mathbf{q}} = qc$$

as a new set of coordinates with the polarization vectors $\varepsilon^{\mu i}$, and identifying

$$n^i_{\mathbf{q}} = C^{i*}_{\mathbf{q}} C^i_{\mathbf{q}} \tag{19}$$

as the equivalent of the photon number operator, we obtain averages by setting

$$\langle n^i_{\mathbf{q}} \rangle = \frac{1}{\exp(\beta\hbar\omega_{\mathbf{q}}) - 1} \tag{20}$$

Then equation (16) reduces to

$$\begin{aligned}
\hat{\Omega}^{\mu\nu}_4(\mathbf{k}) = {} & \frac{\omega_p^2 e^2}{m^2 c^2} \varepsilon^{\mu\alpha\rho}\varepsilon^{\nu\alpha\lambda} B^{0\rho} B^{0\lambda} + \omega_p^4 \left\{ \frac{1}{N}\sum_{\mathbf{q}} \frac{q^\mu q^\nu}{q^2}(S_{\mathbf{k}-\mathbf{q}} - S_{\mathbf{q}}) + \frac{k^\mu k^\nu}{k^2} \right. \\
& + \frac{\beta_p}{\beta r}\left(\delta^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}\right) n(\omega_{\mathbf{k}})\hbar\omega_{\mathbf{k}}\beta_r + \frac{k^2}{\kappa^2}\left[3\frac{k^\mu k^\nu}{k^2} + \left(\delta^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}\right)\right] \\
& + \frac{\beta_p}{\beta_r}\frac{1}{N}\sum_q \left(\delta^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right) S_{\mathbf{k}-\mathbf{q}} n(\omega_q)\hbar\omega_q\beta_r \\
& \left. + \frac{1}{\beta_r mc^2}\sum_q^q \frac{q^\alpha q^\eta}{q^2} \varepsilon^{\mu\alpha\rho}\varepsilon^{\rho\gamma\lambda}\varepsilon^{\nu\alpha\lambda}\varepsilon^{\nu\alpha\delta}\varepsilon^{\delta\eta\alpha} n(\omega_q)\hbar\omega_q\beta_r \right\}
\end{aligned} \tag{21}$$

It is known that in the $k \to 0$ limit no difference should exist between the longitudinal and transverse elements of $\hat{\Omega}^{\mu\nu}_4$. The offending term violating this requirement is

$$\omega_0^4\left[\frac{k^\mu k^\nu}{k^2} + \frac{\beta_p}{\beta_r}\left(\delta^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}\right)\right]$$

We argue that in this limit the distinction between particle (longitudinal) and radiation (transverse) temperatures is meaningless: thus, we treat β_r as a k-dependent quantity, such that $\beta_r(k \to 0) = \beta_p$, while $\beta_r(k \neq 0)$ is not affected by this condition. Since we also consider a nonrelativistic approach in this derivation, the last term is dropped, since it is of the same order as the relativistic corrections. Then equation (21) becomes

$$\hat{\Omega}^{\mu\nu}(\mathbf{k})=\frac{\omega_p^2}{m^2c^2}\,e^{\mu\alpha\rho}\varepsilon^{\nu\alpha\lambda}\beta^{0\rho}\beta^{0\lambda}+\omega_p^4\left\{\delta^{\mu\nu}+\frac{\beta_p}{\beta_r}T_k^{\mu\nu}[f(x_{\mathbf{k}})-1]\right.$$
$$+\frac{k^2}{\kappa^2}(3L_{\mathbf{k}}^{\mu\nu}+T_{\mathbf{k}}^{\mu\nu})+\frac{1}{N}\sum_{\mathbf{q}}L_{\mathbf{q}}^{\mu\nu}(S_{\mathbf{k}-\mathbf{q}}-S_{\mathbf{q}})$$
$$\left.+\frac{\beta_p}{\beta_r}\frac{1}{N}\sum_{\mathbf{q}}T_{\mathbf{q}}^{\mu\nu}S_{\mathbf{k}-\mathbf{q}}f(x_{\mathbf{q}})\right\}\tag{22}$$

where

$$L_{\mathbf{k}}^{\mu\nu}=k^\mu k^\nu/k^2,\qquad T_{\mathbf{k}}^{\mu\nu}=\delta^{\mu\nu}-k^\mu k^\nu/k^2$$
$$x_{\mathbf{k}}=\hbar\omega_{\mathbf{k}}\beta_r,\qquad f(x)=x/(e^x-1)\tag{23}$$

The fourth moment yields

$$\Omega_5^{\mu\nu}(\mathbf{k})=-i\frac{4\pi e^2}{V}\beta_p\frac{d^3}{d\tau^3}\langle j_{\mathbf{k}}^\mu(\tau)\mathbf{j}_{\mathbf{k}}^\mu(0)\rangle\big|_{\tau=0}$$
$$=i\frac{4\pi e^2}{V}\beta_p[\langle\ddot{j}_{\mathbf{k}}^\mu(0)\dot{\mathbf{j}}_{\mathbf{k}}^\nu(0)\rangle-\langle j_{\mathbf{k}}^\mu(0)\dddot{\mathbf{j}}_{\mathbf{k}}^\nu(0)\rangle]$$
$$=i\frac{4\pi e^2}{V}\beta_p\sum_{ij}\langle[\ddot{V}_i^\mu\dot{V}_j^\nu+ik^\alpha V_j^\delta V_j^\nu\ddot{V}_i^\mu-ik^\alpha\dot{V}_i^\alpha\dot{V}_j^\nu V_i^\mu$$
$$+k^\alpha k^\delta\dot{V}_i^\alpha V_i^\mu V_j^\delta V_j^\nu-i2k^\alpha V_i^\alpha\dot{V}_i^\mu\dot{V}_j^\nu$$
$$+2k^\alpha k^\delta V_i^\alpha V_i^\mu V_j^\delta V_j^\nu-k^\alpha k^\gamma V_i^\alpha V_i^\gamma V_i^\mu\dot{V}_j^\nu$$
$$-ik^\alpha k^\gamma V_i^\alpha V_i^\gamma V_i^\mu V_j^\delta V_j^\nu-\dot{V}_i^\mu\ddot{V}_j^\nu+ik^\delta V_i^\delta V_i^\mu\ddot{V}_j^\nu-ik^\alpha\dot{V}_j^\alpha\dot{V}_i^\mu V_j^\mu$$
$$-k^\alpha k^\delta\dot{V}_j^\alpha V_j^\nu V_i^\delta V_i^\mu-i2k^\alpha V_j^\alpha\dot{V}_j^\nu\dot{V}_i^\mu-2k^\alpha k^\delta V_j^\alpha\dot{V}_j^\nu V_i^\mu V_i^\delta$$
$$+k^\alpha k^\gamma V_j^\alpha V_j^\gamma V_j^\nu\dot{V}_i^\mu-ik^\alpha k^\gamma V_j^\alpha V_j^\gamma V_j^\nu V_i^\mu V_i^\delta]\exp[-i\mathbf{k}\cdot(\mathbf{x}_i-\mathbf{x}_j)]\rangle\tag{24}$$

The 16 terms in equation (24) can be evaluated in the same way as those of equation (17). Using the same arguments as those used in solving equation (17), we find that equation (24) becomes

$$\Omega_5^{\mu\nu}(\mathbf{k})=\frac{i\omega_p^2e^3}{2m^3c^3}[2\varepsilon^{\mu\nu\delta}B^{02}B^{0\rho}-\varepsilon^{\mu\delta\rho}B^{0\rho}B^{0\delta}B^{0\nu}-\varepsilon^{\delta\nu\rho}B^{0\rho}B^{0\delta}B^{0\mu}]$$
$$+i\frac{\omega_p^2eB^{0\rho}}{2m^2c\beta_p}\left[(\varepsilon^{\mu\alpha\rho}k^\nu+\varepsilon^{\alpha\nu\rho}k^\mu)k^\alpha+2\varepsilon^{\mu\nu\rho}k^\alpha k^\alpha+(\varepsilon^{\mu\alpha\rho}k^\nu\right.$$
$$\left.+\varepsilon^{\alpha\nu\rho}k^\mu)k^\alpha-\frac{\omega_p^2}{3mc^2\beta_p}\left(2\varepsilon^{\mu\nu\rho}\frac{k^\alpha k^\alpha}{k^2}-\varepsilon^{\mu\alpha\rho}\frac{k^\alpha k^\nu}{k^2}-\varepsilon^{\alpha\nu\rho}\frac{k^\alpha k^\mu}{k^2}\right)\right]$$

$$+\frac{i\omega_p^4 eB^{0\rho}}{2mc}\left\{\frac{1}{N}\sum_{\mathbf{q}}\left(\varepsilon^{\mu\delta\rho}\frac{q^\alpha q^\nu}{q^2}+\varepsilon^{\delta\nu\rho}\frac{q^\delta q^\mu}{q^2}\right)(S_{\mathbf{k}-q}-S_q)+\varepsilon^{\mu\delta\rho}\frac{k^\delta k^\nu}{k^2}\right.$$

$$+\varepsilon^{\delta\nu\rho}\frac{k^\delta k^\mu}{k^2}+\frac{\beta_p}{\beta_r}\left[2\varepsilon^{\mu\nu\rho}+\varepsilon^{\mu\delta\rho}\left(\delta^{\delta\nu}-\frac{k^\delta k^\nu}{k^2}\right)\right.$$

$$\left.+\varepsilon^{\delta\nu\rho}\left(\delta^{\mu\delta}-\frac{k^\mu k^\delta}{k^2}\right)\right]n(\omega_{\mathbf{k}})\omega\hbar\beta_r-\frac{N}{Vmc^2\beta_r}$$

$$\left.\times\sum_{\mathbf{q}}\left[2\varepsilon^{\mu\nu\rho}\frac{q^\alpha q^\alpha}{q^2}-\varepsilon^{\mu\alpha\rho}\frac{q^\alpha q^\nu}{q^2}-\varepsilon^{\alpha\nu\rho}\frac{q^\alpha q^\mu}{q^2}\right]g_{\mathbf{k}-\mathbf{q}}n(\omega_{\mathbf{q}})\hbar\omega_{\mathbf{q}}\beta_r\right\}$$

$$+\frac{i\omega_p^4 e}{2m^2c^3\beta_r}\sum_{\mathbf{q}}\left[2\varepsilon^{\mu\nu\rho}\frac{q^\alpha q^\alpha}{q^2}B^{0\rho}+\frac{q^\alpha q^\rho}{q^2}(\varepsilon^{\rho\nu\alpha}B^{0\mu}\right.$$

$$+\varepsilon^{\mu\alpha\rho}B^{0\nu}-2\varepsilon^{\mu\nu\alpha}B^{0\rho})+2\left(\varepsilon^{\delta\nu\rho}\frac{q^\delta q^\mu}{q^2}B^{0\rho}+\varepsilon^{\mu\delta\rho}B^{0\rho}\right)$$

$$\left.+\frac{N}{V}\left(2\varepsilon^{\mu\nu\rho}\frac{q^\alpha q^\alpha}{q^2}-\varepsilon^{\mu\alpha\rho}\frac{q^\alpha q^\nu}{q^2}-\varepsilon^{\alpha\nu\rho}\frac{q^\alpha q^\mu}{q^2}\right)B^{0\rho}\right]g_{\mathbf{k}-\mathbf{q}}n(\omega_{\mathbf{q}})\hbar\omega_{\mathbf{q}}\beta_r$$

$$+\frac{i\omega_p^4 e\beta_p}{2mNc\beta_r}\sum_{\mathbf{q}}\left[2\varepsilon^{\mu\nu\rho}+\varepsilon^{\mu\delta\rho}\left(\delta^{\delta\nu}-\frac{k^\delta k^\nu}{k^2}\right)+\varepsilon^{\delta\nu\rho}\left(\delta^{\delta\mu}-\frac{k^\delta k^\mu}{k^2}\right)\right]$$

$$\times B^{0\rho}S_{\mathbf{k}-\mathbf{q}}n(\omega_{\mathbf{q}})\hbar\omega_q\beta_r \tag{25}$$

Since we consider a nonrelativistic approach, the relativistic correction terms in equation (25) are negligible compared to unity. Therefore, when terms of the order c^{-2} or higher are dropped in equation (24), we obtain

$$\Omega_5^{\mu\nu}(\mathbf{k})=\frac{i\omega_p^2 e}{2m^2c}(2\varepsilon^{\mu\nu\rho}B^{02}B^{0\rho}-\varepsilon^{\mu\delta\rho}B^{0\rho}B^{0\delta}B^{0\delta}-\varepsilon^{\delta\nu\rho}B^{0\rho}B^{0\delta}B^{0\mu})$$

$$+\frac{i3\omega_p^2 eB^{0\rho}}{m^2c\beta_p}(\varepsilon^{\mu\nu\rho}L_{\mathbf{k}}^{\alpha\alpha}+\varepsilon^{\alpha\mu\rho}L_{\mathbf{k}}^{\alpha\nu}+\varepsilon^{\alpha\nu\rho}L_{\mathbf{k}}^{\alpha\mu})k^2$$

$$+\frac{i\omega_p^4 eB^{0\rho}}{2mc}\left[\varepsilon^{\mu\alpha\rho}L_{\mathbf{k}}^{\alpha\nu}+\varepsilon^{\alpha\nu\rho}L_{\mathbf{k}}^{\alpha\mu}+\frac{\beta_p}{\beta_r}(2\varepsilon^{\mu\nu\rho}+\varepsilon^{\mu\alpha\rho}T_{\mathbf{k}}^{\alpha\nu}\right.$$

$$\left.+\varepsilon^{\alpha\nu\rho}T_{\mathbf{k}}^{\alpha\mu})f(x_{\mathbf{k}})\right]+\frac{i\omega_p^4 eB^{0\rho}}{2mcN}\sum_{\mathbf{q}}\left[(\varepsilon^{\mu\alpha\rho}L_{\mathbf{q}}^{\alpha\nu}+\varepsilon^{\alpha\nu\rho}L_q^{\alpha\mu})(S_{\mathbf{k}-\mathbf{q}}-S_{\mathbf{q}})\right.$$

$$\left.+\frac{\beta_p}{\beta_r}(2\varepsilon^{\mu\nu\rho}+\varepsilon^{\mu\alpha\rho}T_k^{\alpha\nu}+\varepsilon^{\alpha\nu\rho}T_{\mathbf{k}}^{\alpha\mu})S_{\mathbf{k}-\mathbf{q}}f(x_{\mathbf{q}})\right] \tag{26}$$

To obtain an explicit expression for $\Omega^{\mu\nu}_{\iota+1}(k)$, we choose the k-system so that we have

$$\begin{aligned} B^0_x &= B^0_1 = B \sin\theta \\ B^0_y &= B^0_2 = 0 \\ B^0_z &= B_3 = B \cos\theta \end{aligned} \tag{27}$$

and

$$\begin{aligned} q_x &= q^1 = q \sin\theta \cos\Phi \\ q_y &= q^2 = q \sin\theta \cos\Phi \\ q_z &= q^3 = q \cos\theta \end{aligned} \tag{28}$$

3. LONG-WAVELENGTH LIMIT

In the long-wavelength ($k \to 0$) limit we find that the elements of the frequency moments are given by

$$\hat{\Omega}^{11}_2(\mathbf{k}) = \hat{\Omega}^{22}_2(\mathbf{k}) = \hat{\Omega}^{33}_2(\mathbf{k}) = \omega^2_p \tag{29a}$$

$$\hat{\Omega}^{12}_3(\mathbf{k}) = -\hat{\Omega}^{21}_3(\mathbf{k}) = i\omega^2_p\Omega \cos\theta \tag{29b}$$

$$\hat{\Omega}^{23}_4(\mathbf{k}) = -\hat{\Omega}^{32}_3 = i\omega^2_p\Omega \sin\theta \tag{29c}$$

$$\hat{\Omega}^{11}_4(\mathbf{k}) = \omega^4_p\left\{1 + \frac{k^2}{\kappa^2}(1 - \tfrac{2}{15}\beta_p E_{\text{corr}}) + \frac{\beta_p}{\beta^4_r n\hbar^3 c^3}\left[\frac{\pi^2}{45} + \frac{1}{3\pi^2}G_0 + \frac{1}{30\pi^2}(5G_1 + 2G_2)k^2\right]\right\} \tag{29d}$$

$$\hat{\Omega}^{13}_4(\mathbf{k}) = \Omega^{31}_4(\mathbf{k}) = 0 \tag{29e}$$

$$\hat{\Omega}^{22}_4(\mathbf{k}) = \omega^4_p\left\{1 + \frac{k^2}{\kappa^2}(1 - \tfrac{2}{15}\beta_p E_{\text{corr}}) + \frac{\beta_p}{\beta^4_r n\hbar^3 c^3}\left[\frac{\pi^2}{45} + \frac{1}{3\pi^2}G_0 + \frac{1}{3\pi^2}(5G_1 + 2G_2)k^2\right]\right\} \tag{29f}$$

$$\hat{\Omega}^{33}_4(\mathbf{k}) = \omega^4_p\left\{1 + \frac{k^2}{\kappa^2}(3 + \tfrac{4}{15}\beta_p E_{\text{corr}}) + \frac{\beta_p}{\beta^4_r n\hbar^3 c^3}\left[\frac{\pi^2}{45} + \frac{1}{3\pi^2}G_0 + \frac{1}{30\pi^2}(5G_1 + G_2)k^2\right]\right\} \tag{29g}$$

$$\hat{\Omega}^{12}_5(\mathbf{k}) = -\hat{\Omega}^{21}_5(k) = i2\omega^4_p\Omega\left\{1 + \tfrac{1}{2}(3 - \tfrac{2}{15}\beta_p E_{\text{corr}})\frac{k^2}{\kappa^2} + \frac{\beta_p}{2\beta^4_r n\hbar^3 c^3}\left[\frac{\pi^2}{9} + \frac{5}{3\pi^2}G_0 + \frac{1}{30\pi^2}(25G_1 + 9G_2)k^2\right]\right\}\cos\theta \tag{29h}$$

$$\hat{\Omega}_5^{23}(\mathbf{k}) = -\hat{\Omega}_5^{32} = i2\omega_p^4\Omega\left\{1+\tfrac{1}{2}(6+\tfrac{3}{15}\beta_p E_{\text{corr}})\frac{k^2}{\kappa^2} + \frac{\beta_p}{2\beta_r^4 n\hbar^3 c^3}\left[\frac{\pi^2}{9}+\frac{5}{3\pi^2}G_0+\frac{1}{30\pi^2}(25G_1+8G_2)k^2\right]\right\}\sin\theta \tag{29i}$$

where

$$\begin{aligned} E_{\text{corr}} &= \frac{n}{2V}\sum_q \frac{4\pi e^2}{q^2} g_{\mathbf{q}} \\ G_0 &= \int dx\, x^2 f(x) n g_{\mathbf{q}} \\ G_1 &= \int dx\, x^2 f(x) \frac{1}{g}\frac{\partial}{\partial q} n q_{\mathbf{q}} \\ G_2 &= \int dx\, x^2 f(x) \frac{\partial^2}{\partial q^2} n g_{\mathbf{q}} \\ \kappa^2 &= 4\pi e^2 \beta_p n \end{aligned} \tag{30}$$

and $\Omega = e\beta/mc$, the electron cyclotron frequency.

We see that all coefficients of ω^{-2} and ω^{-3} are independent of both correlational and radiation terms. $\hat{\Omega}_4^{13}$ can also be noted as the only coefficient of ω^{-4} that has no correlational and radiation contribution. The correlational terms of $\hat{\Omega}_4^{11}$ and $\hat{\Omega}_4^{22}$ are independent of magnetic field and are the same as in the nonmagnetized case (Kalman and Genga, 1986), i.e.,

$$-\tfrac{2}{15}\omega_p^4\beta_p E_{\text{corr}}\frac{k^2}{K^2} \tag{31}$$

The correlational term of $\hat{\Omega}_4^{33}$ is also independent of magnetic field and is equal to

$$\tfrac{4}{15}\omega_p^4\beta_p E_{\text{corr}}\frac{k^2}{K^2} \tag{32}$$

which is the same result obtained in the magnetic field-free case (Kalman and Genga, 1986). The correlational terms of $\hat{\Omega}_5^{12}$ and $\hat{\Omega}_5^{23}$ are imaginary and magnetic field-dependent; they are of the form

$$-i\tfrac{2}{15}\omega_p^4\Omega\beta_p E_{\text{corr}}\frac{k^2}{\kappa^2} \tag{33}$$

and

$$i\tfrac{3}{15}\omega_p^4\Omega\beta_p E_{\text{corr}}\frac{k^2}{\kappa^2} \tag{34}$$

respectively, where E_{corr} is the (negative) correlation energy per particle. It can also be noted that the correlational terms of Ω_4^{11}, Ω_4^{22}, and Ω_5^{12} have opposite signs to those of Ω_4^{33} and Ω_5^{23}. The effect of finite radiation temperature is manifested through G_0, G_1, and G_2 and is very small under normal conditions, i.e., in laboratory plasmas, but cannot be ignored in situations where the radiation temperature is very high; for instance, in stellar interiors. We also find that radiation contributions are of the order T_r^4 and not T_r^8, which would be the expectation on the basis of the naive radiation drag model (Landau and Lifshitz, 1960).

4. STRONG COUPLING AND FINITE-RADIATION-TEMPERATURE EFFECTS ON PLASMA DISPERSION

Although the high-frequency sum rule is exact, it requires that $\Omega\omega^{-1} \ll 1$, $\omega_p\omega^{-1} \ll 1$; thus, it is not very reliable for the calculation of the dispersion relations. Under normal conditions the effects of finite radiation temperature are very small and can be ignored. However, in situations where the finite radiation temperature is very high, as in the case of stellar interiors, its contribution must be taken into account. The high-frequency modes of interest are the ordinary and the extraordinary modes. The extraordinary mode of interest is the one with cutoff frequency

$$\omega_2 = \tfrac{1}{2}\Omega[1 + (1 + 4\omega_p^2/\Omega^2)^{1/2}]$$

We consider all the modes for propagation parallel and perpendicular to the magnetic field. We use a coordinate system where $\mathbf{k} = (0, 0, k)$ and $\mathbf{B}^0$ is the $\mathbf{x} - \mathbf{z}$ plane, i.e., the k-system.

We determine the properties of the high-frequency waves by applying a small perturbation on the dispersion; the shift of frequency due to correlational and finite-radiation-temperature effects occurs as a result of this. The correlation is very weak for weakly coupled plasmas, but can be strong for strongly coupled plasmas. The frequency shift due to correlation is of order k^2, and thus is small as $k \to 0$, which is equal to the order of the frequency shifts caused by refractive, thermal, and quantum effects, respectively, even for $\gamma \gg 1$. That is, after perturbation we find that

$$\omega = \omega^0 + \delta\omega; \qquad \delta\omega \ll \omega^0 \tag{35}$$

where

$$\omega^0 = \begin{cases} \omega_p: & \text{ordinary mode} \\ \omega_2: & \text{high-frequency extraordinary mode} \end{cases} \tag{36}$$

$$\delta\omega = -\frac{\Delta_1(\gamma, \beta_r; k, \omega)}{\Delta_0'(\omega)} \tag{37}$$

with

$$\Delta_0'(\omega) = \left.\frac{\partial \Delta_0(\omega)}{\partial \omega}\right|_{\omega=\omega^0} \tag{38}$$

The frequency shift $\delta\omega$ is due to refractive, thermal, correlational, and finite-radiation-temperature effects; equation (35) is obtained by applying a Taylor expansion to the dispersion relation $\Delta = 0$ about ω^0, where Δ is a function of the components of the dielectric tensor.

4.1. Propagation Parallel to Magnetic Field

In this case the dispersion relation is given by

$$[(\varepsilon_{11} - n^2)^2 - \varepsilon_{12}^2]\varepsilon_{33} = 0 \tag{39}$$

which leads to

$$\varepsilon_{33} = 0 \tag{40}$$

for the longitudinal mode and

$$(\varepsilon_{11} - n^2)^2 - \varepsilon_{12}^2 = 0 \tag{41}$$

for the extraordinary mode. We find that in this situation the ordinary mode does not exist; instead, we have the longitudinal mode.

4.1.1. Longitudinal Mode

Since the longitudinal mode oscillates at the plasma frequency, after applying a small perturbation to the dispersion relation, we find that

$$\delta\omega = \frac{\omega_p}{2}\left[C_L(\gamma, \beta_r) + A_L(\gamma, \beta_r)\frac{k^2}{\kappa^2}\right] \tag{42}$$

where

$$A_L(\gamma, \beta_r) = 3 + \tfrac{4}{15}\beta_r E_{\text{corr}} + \frac{1}{30\pi^2}\frac{\beta_p \kappa^2}{\beta_r^4 n\hbar^3 c^3}(5G_1 + G_2) \tag{43}$$

$$C_L(\gamma, \beta_r) = \frac{\beta_p}{\beta_r^4 n\hbar^3 c^3}\left(\frac{\pi^2}{45} + \frac{1}{3\pi^2}G_0\right) \tag{44}$$

For $\beta_r^{-1} \to 0$, A_L is known (Kalman and Golden, 1979) to change from positive to negative values for $\gamma > \gamma_{\text{crit}}$. From both molecular dynamics computer results (Hansen *et al.*, 1974, 1975; Baus and Hansen, 1980) and recent, more sophisticated theoretical results (Golden and De-Xin Lu, 1985) indicate an actual value of $\gamma_{\text{crit}} \cong 45$, but according to equation (43), $\gamma_{\text{crit}} > \frac{45}{2}$. The effect of finite radiation temperature is manifested through G_0, g_1, and

G_2; for all situations but the combined occurrence of strong enough coupling to induce oscillations in g_q and extremely high radiation temperature causing the photon thermal wavelength to become shorter than the interparticle spacing, it is expected that $G_0 > 0$, while $G_1 > 0$, $g_2 > 0$; however, even for $G_0 < 0$, the $\pi^2/45$ term is expected to dominate. Thus, a finite radiation temperature results in (1) an upward renormalization of the plasma frequency from

$$\omega_p \quad \text{to} \quad \omega_p(1 + C_L/2) \tag{45}$$

and (2) a reduction of the negative correlational effect on plasmon dispersion for finite k.

4.1.2. Extraordinary Mode

In this case we find that

$$\delta\omega = \frac{\omega_p^2}{2\omega_2}\left\{C_L(\gamma, \beta_r) + \frac{\omega_2^2}{\omega_p^4}\left[\left[1 + \frac{\omega^2}{\Omega} C_L(\gamma, \beta_r)\right]c^2 \right.\right.$$
$$\left.\left. + \frac{\omega_p^4}{\omega_2^2} A_T(\gamma, \beta_r)\right]k^2\right\} \tag{46}$$

where

$$A_T(\gamma, \beta_r) = 1 - \tfrac{2}{15}\beta_p E_{\text{corr}} + \frac{\beta_p \kappa^2 \hbar^{-3}}{30\pi^2 \beta_r^4 n}(5G_1 + 2G_2) \tag{47}$$

If C_L is very large, then a finite radiation temperature results in the upward renormalization of the extraordinary frequency from

$$\omega_2 \quad \text{to} \quad \omega_2\left(1 + \frac{\omega_p^2}{2\omega_2^2} C_L\right) \tag{48}$$

Further, it can be seen that finite-radiation-temperature and correlational effects enhance the dispersion for finite k.

4.2. Propagation Perpendicular to Magnetic Field

The dispersion relation is given by

$$(\varepsilon_{11} - n^2)[(\varepsilon_{22} - n^2)\varepsilon_{33} - \varepsilon_{23}^2] = 0 \tag{49}$$

from which we have

$$\varepsilon_{11} - n^2 = 0 \tag{50}$$

for the pure transverse mode, which is called the "ordinary mode" because the propagation is not affected by the external magnetic field, and

$$(\varepsilon_{22} - n^2)\varepsilon_{33} - \varepsilon_{23}^2 = 0 \tag{51}$$

for the coupled transverse-longitudinal mode, which is called the "extraordinary mode" since the propagation is affected by the external magnetic field.

4.2.1. *Ordinary Mode*

The frequency shift is given by

$$\delta\omega = \frac{\omega}{2}\left\{ C_L(\gamma_1\beta_r) + \left[\frac{c^2}{\omega_p^2} + A_T(\gamma_1\beta_r)\right]k^2\right\} \tag{52}$$

This shows that a finite radiation temperature results in the upward renormalization of the ordinary mode from ω_p to $\omega_p(1+\frac{1}{2}C_L)$ as is the case of the longitudinal mode above. The correlations are seen to increase, rather than decrease, the positive thermal effect, in contrast to the effect of correlations on longitudinal plasmon dispersion for finite k. If one argues that the effects on dispersion can be seen in the system for strong coupling in an attempt to emulate the mode structure of a Wigner lattice, this result is not surprising, since the high-frequency transverse phonons, in contrast to the longitudunal ones, do exhibit a positive dispersion (Kalman, 1978).

4.2.2. *Extraordinary Mode*

In this case we have

$$\delta\omega = \frac{\omega_p^2}{2\omega_2}\left\{ C_L(\gamma, \beta_r) + \frac{\omega_2}{2\omega_p^4}\left[1 + c^2 + \frac{2\omega_p^4}{\omega_2^2}\frac{A_x}{k^2}(\gamma, \beta_r)\right]k^2\right\} \tag{53}$$

where

$$A_x(\gamma, \beta_r r) = 2 + \tfrac{1}{15}\beta_p E_{\text{corr}} + \frac{\beta_p \kappa^2}{60\pi^2\beta_r^4 n\hbar^3 c^3}(10G_1 + 3G_2) \tag{54}$$

It can be seen that the finite-radiation-temperature effect results in an upward renormalization of the extraordinary frequency from ω_2 to $\omega_2[1+\frac{1}{2}(\omega_p^2/\omega_2^2)C_L]$ for large C_L; it also reduces the negative correlational effect on thermal dispersion for finite k.

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